XI. *Determination of the Number of Electrostatic Units in the Electromagnetic Unit made in the Physical Laboratory of Glasgow University.* By DUGALD M'KICHAN, *M.A.* *Communicated by Sir* WILLIAM THOMSON, *F.R.S. &c.*

Received April 15,—Read May 15, 1873.

THE object of the following paper is to describe observations made to determine the relation between the fundamental units in the electromagnetic and electrostatic systems of absolute measurement.

These observations were made at intervals from 1870 to 1872, first in the Physical Laboratory of the Old College, Glasgow, and afterwards in that of the New College.

In the former series of experiments I had the advantage of the assistance of the late WILLIAM LEITCH, of Glasgow*. The latter series I made chiefly with the assistance of GEORGE A. HILL, M.A., now Assistant Professor of Natural Philosophy in Harvard College, U. S.

A summary is also given of the results of similar experiments made by Mr. WILLIAM F. KING in the Old College in 1867 and 1868.

The two systems of absolute electrical measurement are founded on the fundamental units of time, mass, and space applied to the observed effects of electricity at rest and electricity in motion, the one system having a definite numerical relation to the other. The number of electrostatic units in the electromagnetic unit of quantity, from the dimensions of the unit of quantity in the two systems, is expressible as a velocity, and is usually designated by the symbol v.

In the Report of the British Association's Committee on Electrical Measurements, 1863, five possible methods are enumerated for experimentally determining v. It is necessary and sufficient that we should obtain a common electromagnetic and electrostatic measure of some electric quantity, current, resistance, electromotive force, or capacity.

The first numerical determination of v had been made by WEBER and KOHLRAUSCH†, who had employed the method last indicated: they derived the value of v from a common measure of capacity. The capacity of a condenser was measured electrostatically by comparison with the capacity of a sphere of known radius, and electromagnetically by passing the discharge from the condenser through a galvanometer. The value of v derived from their experiments was $310{\cdot}74\times10^8$ centimetres per second.

* Mr. LEITCH afterwards went out to introduce Sir WILLIAM THOMSON'S Siphon-recorder at the various stations of the Eastern Telegraphs. He died on his way home, at Alexandria, after he had completed his work.

† POGG. Ann., Aug. 1856, Band xcix. p. 10.

Professor CLERK MAXWELL has pointed out* a probable source of error in these experiments of MM. WEBER and KOHLRAUSCH. A neglect, in the measurement of the capacity of the condenser, of the phenomenon of "electric absorption," the nature of which was not then well understood, would lead to an overestimation of the electrostatic capacity of the condenser, and consequently to a value of v which would be too great.

In the same paper Professor CLERK MAXWELL describes a direct comparison of electrostatic with electromagnetic force in which no absolute measurement of either was made. The ratio of electrostatic and electromagnetic units of quantity was determined by a direct experiment, in terms of the British-Association Unit of Resistance, by a balance of the electrostatic attraction of two disks maintained at a great difference of potentials and the electromagnetic repulsion of two circular coils attached to the attracting disks. The mean value of v derived from this comparison was $288{\cdot}0 \times 10^8$ centims. per second.

In the experiments to be described, the method employed was that of obtaining a common measure of electromotive force. The electromotive force produced by a battery can be measured directly in absolute electrostatic units; its electromotive force can also be obtained in electromagnetic measure from the current which it produces in a circuit of known resistance.

The measure of electromotive force from one point to another being defined (in the two systems) as the work done by electrical forces during the passage of unit quantity of electricity from the one to the other, v, the ratio of the units of quantity in the two systems, is also the inverse ratio of the units of electromotive force.

For this comparison three independent measurements are required—the determination of the resistance of the given circuit in absolute measure, the electrostatic measure of electromotive force produced by a given battery, and the electromagnetic measure of the current maintained in the circuit by this electromotive force.

The resistance of the circuit was obtained, according to the usual method, from a comparison with resistance-coils derived from standards the resistance of which had been experimentally determined in absolute measure by observations completed in 1864 at King's College, for the Committee of the British Association, by Professors J. CLERK MAXWELL, BALFOUR STEWART, and FLEEMING JENKIN. Accordingly, whatever corrections may hereafter be discovered to be applicable to the absolute value of the unit of resistance must be applied to the value of v here determined. It is hoped that Professor MAXWELL will soon be able to accomplish the new determination of the absolute resistance of the standard coil which he proposes to make in the Cavendish Laboratory, Cambridge University.

The electromotive force of the battery, which consisted of a series of DANIELL'S cells, was measured electrostatically or weighed by means of Sir WILLIAM THOMSON'S New Absolute Electrometer. A description of this instrument will be found in a "Reprint of Papers on Electrostatics and Magnetism" by Sir WILLIAM THOMSON, §§ 364–367.

* Philosophical Transactions, 1868, p. 651.

The New Absolute Electrometer is identical in principle with the Absolute Electrometer described in the Report of the British Association, 1867. The principal improvements in the new instrument are the introduction of a new sighting arrangement, by which errors from parallax are avoided, the substitution of a metal disk supported by springs for the disk balanced by a counterpoise, and the addition of an idiostatic gauge and a replenisher, by means of which the charged disk can be maintained at a constant potential. By this instrument the force of attraction between two disks of known area at a constant difference of potentials is measured in grammes weight. The difference of potentials between the two poles of a battery can thus be measured, and the area, force, and distance being known, can be expressed in absolute electrostatic units of space, time, and mass. In these measurements the units employed were:—for unit of mass the gramme, for unit of time the second, and for unit of space the centimetre.

The strength of the current was measured by the force acting between two portions of itself in the circuit of an electrodynamometer (see Plate XXXII. figs. 1 & 2). This circuit consisted of three coils, two large fixed coils (a, b), and a third smaller coil (c) movable round a vertical axis. The larger coils contained each upwards of 3700 metres of fine silk-covered wire wound upon brass rings, 30 centims. in diameter, with parallel flanges about $2\frac{1}{2}$ centims. broad.

The two coils contained on an average 3700 turns of wire each, the depth of the coils being somewhat less than 2 centims. and the breadth of the rings being 2 centims. One end of the wire in each, before winding, was soldered to the ring, the ring thus forming one of the terminals of the coil. The two coils were placed on an oblong wooden pedestal (h) (supported on three screws) with their planes vertical and parallel to one another, the line joining their centres being perpendicular to the plane of each coil. The rings were fitted to wooden concave supports (g, g'), to which they were secured by two bars of wood (f, f') resting across the rings on the inside, and firmly attached by means of screws to the wooden stand. The parallelism of the coils was preserved, as far as possible, by means of stout hard-wood bars of equal length (i) inserted at intervals between the rings perpendicular to the planes of the coils. To secure the most uniform magnetic field in the neighbourhood of the coils, the HELMHOLTZ arrangement of the coils was chosen, the distance from centre to centre of the coils being approximately equal to their radius.

The movable or suspended coil (c) consisted of upwards of 900 metres of finer wire. The brass ring upon which it was wound was $6\frac{1}{2}$ centims. in diameter, and had flanges $2\frac{1}{2}$ centims. broad. It contained upwards of 3000 turns of wire, the depth of the coil being 2 centims. and its thickness $1\frac{1}{2}$ centim.

This movable coil, which was suspended between the two fixed coils, was protected from disturbing currents of air by a wooden box (k) which stood between the two fixed coils, being fastened to the stand on which they rested. On one side perpendicular to the planes of the coils this box was left open; and after the coil had been suspended, a glass screen was introduced. Subsequently, to prevent any error from distortion of the rays of light passing through the glass, and to avoid any unnecessary diminution of their brightness, the glass was removed and sheets of cardboard, with apertures through which the rays might pass, were substituted. Two or three such screens (*l*), their apertures varying in size with the distance of each screen from the coil, formed a sufficient protection to the coil from currents of air. The box was large enough to admit of the free motion of the coil through a considerable angle; and for temporary adjustments the coil was accessible from behind, an aperture being left which was closed while observations were being made, but which could be opened when necessary.

The coil was not suspended from the roof of the box, which did not stand so high as the outside coils; but a higher point of suspension was found by erecting a brass vertical tube (*d*), about 2 centims. in diameter and 35 centims. in length, over an aperture in the roof of the box. The suspending-wire was attached to the cover (*e*) which closed this tube. This cover or lid, which fitted into the mouth of the tube, being movable, the position of the coil in azimuth was easily adjustable. The suspending-wire, which acted also as a conducting-wire, was soldered to a cross piece fixed to the flanges of the ring at right angles to them: this wire, being thus in metallic connexion with the ring to which one end of the insulated wire was soldered as in the fixed coils, served as one of the terminals of the suspended coil. The other end of the insulated wire was soldered to a brass terminal set in a cross piece of vulcanite at the bottom of the coil: to this terminal a long very fine platinum wire was attached, its other end being soldered to a stout wire fastened to the bottom of the box, and leading out to the other connexions.

The movable coil was suspended symmetrically with respect to the fixed coils. Its centre was accurately marked on a piece of wood temporarily inserted inside the ring. The coil was then raised by means of the suspending-wire until this centre coincided with a fine wire temporarily fixed in the line joining the centres of the fixed coils.

Again, by altering the position of the vertical tube relatively to the box to which it was fastened, the fixed coils being kept vertical, the vertical axis of the coil was adjusted to be equidistant from the planes of the fixed coils.

The three coils just described were joined in series, the connexions being so arranged that a current passing through the three coils should pass in the same direction through the two fixed coils so that they should conspire in their action on the movable coil.

The strength of the current was measured by the deflection of the suspended coil. In order that this determination, by means of the deflection, might be absolute, it was necessary to eliminate the effect due to the horizontal component of the earth's magnetism. This was done by observing the deflections when a given current passed first in the one direction and then in the other, the relations between the coils themselves remaining fixed. The mean deflection was that due to the simple action of the coils. To render the discrepance between the two deflections as small as possible, magnets were fixed in the neighbourhood of the coils to neutralize the action of terrestrial magnetism, and the deflections due to equal and opposite currents were as nearly as possible equalized.

If it had been possible to adjust the magnets so as exactly to counterbalance the directive component of the earth's magnetism, these deflections would have been exactly equal; but a slight discrepance between the deflections remained after the adjustment of the magnets. This remaining discrepance was eliminated in the calculation by taking the mean between the two deflections.

These deflections, which were measured according to the usual method by reflection, were read off on a scale (Plate XXXII. fig. 3, *f*) (divided to fortieths of an inch) placed at a distance of 3 or 4 metres from the coil. The light (from a gas-lamp constructed like a paraffin-lamp with glass shade &c. supported on a wooden frame attached to the table on which the electrodynamometer stood) was reflected from a light circular mirror (*m*) cemented to the lower part of the front of the movable coil. The lamp (*a*) was placed about midway between the electrodynamometer and the scale, the reflected rays passing under it from the mirror away to the scale. The rays reached the mirror after passing through a condensing-lens (*c*) attached to a long telescope-tube (*b*), the position of which, relatively to the lamp and mirror, was adjusted by moving the tripod stand (*d*) to which its supports were attached. The lamp carried a metal screen with slit and fine wire stretched across the slit; and the deflections were noted, as in reflecting galvanometers and electrometers, from the positions of the shadow of this wire in the reflected image.

To render these deflections available for absolute measurement, it was necessary to know in mechanical units the force required to maintain the coil deflected through any given angle. This force is given by the equation of oscillations in terms of the period of vibration and the moment of inertia of the coil round its vertical axis. The period of vibration of the coil was determined from a series of 110 periods. The times of the first 10 and the last 10 having been noted, and the differences of the times of the 1st and the 101st, the 2nd and the 102nd, &c. having been taken, the average period of vibration for the whole series was very accurately determined. As the deflections to be observed in the actual measurements were made in a magnetic field with the coil deflected from its zero-position, these observations of the period were repeated with the coil deflected under the influence of the current about to be used in the comparison. A result differing by nearly $\frac{1}{2}$ per cent. was sometimes obtained. This determination of the period of vibration of the coil was generally repeated before any lengthened series of observations.

The moment of inertia of the coil round its vertical axis was obtained from a comparison of its vibrations with those of a symmetrical body of the same weight vibrating under the same conditions. The coil having been carefully weighed, a vibrator of the same weight was constructed. The vibrator consisted of a cylindrical brass ring (radius 5 centims.) and a light brass bar attached at its extremities to the ring. This was made to vibrate, in an enclosed space protected from currents of air, round an axis perpendicular to the plane of the ring, the suspending-wire being attached to the centre of the cross bar. The image of a light reflected from a galvanometer-mirror fixed to the vibrator at its axis allowed these vibrations to be easily counted. The period of vibration was determined from a series of oscillations extending through 110 periods. The vibrator having been removed from the suspending-wire, the coil was substituted in its place (care having been taken that the wire was not subjected to any stretching in the interval); and a similar series of observations was made to determine the period of vibration of the coil. The moments of inertia of any two bodies of equal weight around any axis being proportional to the squares of their periods of vibration round these axes, the ratio between the moment of inertia of the suspended coil and the moment of inertia of the ring-vibrator was given by this comparison.

The dimensions of the vibrator were accurately known, and the summation was made for all its parts. This, together with the ratio just obtained, gave the moment of inertia of the movable coil round a vertical axis through its centre in centimetre-grammes. This moment of inertia, combined with the period of vibration of the coil after it was suspended in the electrodynamometer, gave the absolute measure of the couple required to hold the coil deflected through unit-angle or any fraction of unit-angle. The length of a scale-division and the distance of the scale from the mirror being known, the angular value of a deflection through any number of scale-divisions was known, and consequently the deflecting couple corresponding to each reading on the electrodynamometer scale.

The electromagnetic measure of this couple was obtained in terms of the strength of the deflecting current from a consideration of the number of turns of wire in the coils and the area they enclosed, the distance of the movable coil from the fixed coils, and other quantities entering into the equation below. These two values, the mechanical and the electromagnetic (which for any given deflection must be equal to one another), having been equated, the strength of the current was obtained in mechanical or absolute electromagnetic units, the unit strength of current being defined, in a purely mechanical way, as "that current of which the unit of length placed along the circumference of a circle of unit radius produces a unit of magnetic force at the centre."

The methods of obtaining the corresponding absolute measurement of the resistance of the circuit and the electrostatic measure of the electromotive force have been already mentioned. These three measurements afford the data from which the ratio v is to be obtained.

Mathematical Theory of the Comparison.

I. Absolute Electrometer (see "Reprint" above referred to, § 362).

Let F=force (determined by experiment) required to displace the movable disk of the electrometer through a known distance.

A=mean of the areas of the disk and the aperture of the guard-ring which surrounds it.

V=difference of potentials between the disk and the opposed plate connected with one of the poles of the battery.

V'=difference of potentials between the disk and the opposed plate connected with the other pole of the battery.

D=distance through which the opposed plate has to be moved from its zero position to bring the disk again into the sighted position after one of the poles of the battery has been connected with the opposed plate.

D'=distance through which the opposed plate has to be moved from its zero position to bring the disk again into the sighted position after the other pole of the battery has been connected with the opposed plate.

Then $V=D\sqrt{\frac{8\pi F}{A}}$,

$$V'=D'\sqrt{\frac{8\pi F}{A}},$$

$$\frac{V-V'}{2}=\frac{D-D'}{2}\cdot\sqrt{\frac{8\pi F}{A}}=\text{difference of potentials between the two poles of the battery.}$$

It was not necessary to obtain D and D' separately, $D-D'$ being given in the difference between the two electrometer readings got by connecting first one pole of the battery and then the other with the lower plate of the electrometer.

The mechanical displacement due to F was determined by a long series of experiments, in which a known weight was distributed over the disk and the displacement of the disk observed. The average result of the later weighings was that 200·55 micrometer-divisions represented the displacement due to a force of ·6 gramme weight. A similar series of observations in the Laboratory of the Old College gave 200·6 as the average displacement. In the comparisons then made 200·6 was employed, but the difference being only that of $\frac{1}{40}$ per cent. is inconsiderable.

II. Electrodynamometer.

Let n, n'=number of turns on the two fixed coils respectively.

r, r'=mean radii of the fixed coils.

b=distance of the mean plane of the fixed coils from the centre of the movable coil, or half the distance of the mean planes from one another.

C=strength of current in the coils.

$2\pi nr$ and $2\pi n'r'$ are the effective lengths of wire in the coils.

Let α and α'=angles subtended at the centre of the movable coil by the radii of the fixed coils.

Then the effective component of the magnetic force at the centre of the movable coil due to the fixed coils is

$$\frac{2\pi nrC}{r^2+b^2}\sin\alpha+\frac{2\pi n'r'C}{r'^2+b^2}\sin\alpha'=C\left\{\frac{2\pi nr^2}{(r^2+b^2)^{\frac{3}{2}}}+\frac{2\pi n'r'^2}{(r'^2+b^2)^{\frac{3}{2}}}\right\}.$$

For each of the coils the following correction for breadth and depth of the coils given by Professor CLERK MAXWELL* is to be employed.

* Electricity, Art. 700.

Let ξ, ξ'=depth of the section of the coils in the plane of the coils,

η, η'=breadth of section of coils perpendicular to the plane of the coils,

then the effective component of the force due to one of the coils is

$$C.\frac{2\pi nr^2}{(r^2+b^2)^{\frac{3}{2}}}\left\{1+\tfrac{1}{24}\frac{\xi^2}{r^2}(2-15\sin^2\alpha\cos^2\alpha)+\tfrac{1}{24}\frac{\eta^2}{r^2}(15\sin^2\alpha\cos^2\alpha-3\sin^2\alpha)\right\},$$

and that due to the other

$$C.\frac{2\pi nr'^2}{(r'^2+b^2)^{\frac{3}{2}}}\left\{1+\tfrac{1}{24}\frac{\xi'^2}{r'^2}(2-15\sin^2\alpha'\cos^2\alpha')+\tfrac{1}{24}\frac{\eta'^2}{r'^2}(15\sin^2\alpha'\cos^2\alpha'-3\sin^2\alpha')\right\}.$$

For the left coil,

outer radius=16 902 centims., inner radius=15·312 centims.,

r=16·11 centims., ξ=1·59 centim., η=2·00 centims.,

$\sin\alpha=\frac{r}{(r^2+b^2)^{\frac{3}{2}}}$, $\cos\alpha=\frac{b}{(r^2+b^2)^{\frac{3}{2}}}$, and the correcting factor=·9998234.

For the right coil,

r'=16·24 centims., ξ'=1·856 centim., η'=1·95 centim., and the correcting factor =·9997692.

Calling these factors F and F′, we have for the magnetic force at the centre of the suspended coil due to the two fixed coils when a current of strength C is passing through them,

$$\left[\left\{\frac{2\pi nr^2}{(r^2+b^2)^{\frac{3}{2}}}\right\}F+\left\{\frac{2\pi n'r'^2}{(r'^2+b^2)^{\frac{3}{2}}}\right\}F'\right]C. \quad \text{(1)}$$

The magnetic moment of the suspended coil is equal to the product of the strength of current and the sum of all the areas enclosed by the wire.

Let c=a mean radius of the suspended coil, i=number of turns of wire in the suspended coil,

then $C.i\pi c^2$=its magnetic moment, and the couple due to a current C tending to turn the suspended coil round a vertical axis through its centre is

$$\left[\left\{\frac{2\pi nr^2}{(r^2+b^2)^{\frac{3}{2}}}\right\}F+\left\{\frac{2\pi n'r'^2}{(r'^2+b^2)^{\frac{3}{2}}}\right\}F'\right]C^2 i\pi c^2. \quad \text{(2)}$$

The outer radius of the suspended coil was 5·240 centims., and the inner radius 3·259 centims. The difference of radii, 1·981 centim., being considerable in comparison with the radii themselves, the quantity $i\pi c^2$ was determined experimentally, after the observations had been completed, by a method to be afterwards described. [The result then obtained is used in all the calculated values of v, except those of 1867 and 1868.] The mechanical value of the deflecting couple (2) is given by the equation $\frac{d^2\theta}{dt^2}+\mu\theta=0$, in which $\mu=\frac{E}{Wk^2}$, Wk^2 being the moment of inertia of the coil round a vertical axis, and E the elastic couple per unit angle.

The solution of this equation gives (T=period of vibration of the coil)

$$T=\frac{2\pi}{\sqrt{\mu}}=2\pi\sqrt{\frac{Wk^2}{E}}, \text{ and } E=\frac{4\pi^2 Wk^2}{T^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Let γ=strength of current which gives a deflection of one scale-division.

The deflection of the reflected image being twice the deflection of the coil, the elastic couple corresponding to a deflection of one scale-division is

$$E.\frac{\text{length of 1 scale-division}}{\text{twice distance of scale}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Equating (2) and (4), we have

$$\frac{4\pi^2 Wk^2}{T^2}\times\frac{\text{length of scale-division}}{2\times\text{distance of scale}}=\left[\left\{\frac{2\pi nr^2}{(r^2+b^2)^{\frac{3}{2}}}\right\}F+\left\{\frac{2\pi n'r'^2}{(r'^2+b^2)^{\frac{3}{2}}}\right\}F'\right]i\pi c^2\gamma^2. \quad . \quad . \quad (5)$$

Equation (5) gives γ in terms of units of space, time, and mass. It is seen also from this equation that the deflection produced by a current passing through the coils of the electrodynamometer is proportional to the square of the strength of the current. Accordingly the strength of current indicated by any given deflection is obtained by multiplying the value of γ given in (5) by the square root of the number of scale-divisions in the deflection.

Results of the Comparison.

I. The following is a summary of the results obtained by Mr. KING, from a comparison of measurements with the Electrodynamometer and the (old) Absolute Electrometer:—

		centims. per second.
1867. Nov. 18–27 . .	Electrodynamometer compared with Standard Electrometer, which had been previously compared with the Absolute Electrometer (indirect)	$v=297\cdot2\times10^8$
Dec. 3–20 . .	Direct Comparison of Electrodynamometer and Absolute Electrometer	$v=289\cdot2\times10^8$
1868. Mar. 10–Apr. 7 .	Direct Comparison	$v=284\cdot5\times10^8$
Apr. 20–30 . .	Direct Comparison; observers and all the instruments insulated	$v=283\cdot7\times10^8$
May 1–10 . .	Direct Comparison	$v=285\cdot6\times10^8$

Mean value of v given by this comparison, leaving out the two first sets (which were taken under very unfavourable circumstances), is $284\cdot6\times10^8$ centims. per second.

II. The comparisons which were made during March 1870 in the Laboratory of the old College were preceded by some alterations in the electrodynamometer, and a fresh determination of several of the quantities connected with it. Resistances were measured by a new set of standard coils, and the old absolute electrometer was superseded, in the comparison, by the new absolute electrometer already referred to.

The movable coil was resuspended, and a light mirror, the opposite sides of which had been ground parallel, was substituted for the heavier mirror that had been formerly used.

The moment of inertia of the vibrator with which the coil was compared was 4307·498 centimetre-grammes.

The periods of vibration of coil and vibrator similarly suspended were 4·69 and 7·795 seconds respectively. Whence for the coil

$Wk^2=4307{\cdot}498\times(\frac{4{\cdot}69}{7{\cdot}795})^2=1559{\cdot}3$ centimetre-grammes.

Addition for cross piece $=\quad 0{\cdot}2$ „ „

Total moment of inertia $=1559{\cdot}5$ „ „

Period of vibration of coil after being suspended in the electrodynamometer was $T=4{\cdot}800$ seconds.

Distance of scale $=446{\cdot}5$ centims.

Length of each scale-division $=\frac{1}{40}$ inch $=\frac{2{\cdot}539954}{40}$ centims.

Number of turns on left-hand coil, $n=3720{\cdot}75$.

Number of turns on right-hand coil, $n'=3668{\cdot}50$.

Mean radius of left-hand coil, $r=16{\cdot}11$ centims.

Mean radius of right-hand coil, $r'=16{\cdot}24$ centims.

Distance of mean plane of coils from centre of the suspended coil, $b=8{\cdot}185$ centims.

$i\pi c^2$ (determined by experiment May 10, 1872) $=195{,}420$ sq. centims.

These values being substituted in equation (5), for this series of observations $\gamma={\cdot}000021842$ electromagnetic units.

The following Tables exhibit results obtained from simultaneous measurements of current and electromotive force. It would have been desirable to have had more frequent measurements of the resistance of the coils; but as only two observers were generally available, and the comparison of the electrodynamometer and electrometer required the attention of two observers, this was not generally practicable. There was always a liability in the coils to a variation in resistance, due to the variation in the temperature of the coils caused by the current; but as the current was not allowed to pass through the coils for a longer time than was just necessary for its measurement, these variations were not considerable.

The plan of the connexions is indicated in Plate XXXII. fig. 4. Two terminals were led from the battery (B) to a reversing key (K). The wires leading from the key were connected one to the end of the left-hand coil (A′), the other to the opposite end of the right-hand coil (A). From these points, where the battery joined the electrodynamometer coils, electrodes were brought to another reversing key (C) which was connected with the absolute electrometer (E), one of its wires being connected with the insulated electrode of the electrometer which communicated with the insulated lower plate, the other being in contact with the outside coating of the jar of the electrometer. The difference of potentials thus measured was not the entire difference of potentials between the two poles of the battery, but the reduced difference maintained between the extremities of the electrodynamometer circuit.

When two deflections, with the current passing in opposite directions, had been read off on the electrodynamometer scale, the mean between these two deflections was taken as representing what would have been the deflection supposing that the process of eliminating the horizontal component of the earth's magnetism had been perfectly successful. These deflections are noted as R. and L. (right and left) with reference to the sides of the battery-key. Corresponding to each, two readings were obtained in the absolute electrometer by reversing the electrometer-key while the battery-circuit continued "made," half the difference of readings due to this reversal representing the difference of potentials between the two ends of the coils through which the current was passing.

It was found, however, in most of the comparisons, that the current corresponding to a difference of potentials sufficiently large to be measured on the electrometer with a small percentage of error, produced a deflection in the electrodynamometer beyond the limits of the scale.

In order to reduce the strength of current without at the same time reducing the difference of potentials to be measured by the electrometer, a large resistance (R), generally equal to that of the circuit formed by the coils, was introduced between the coils and the battery, and the difference of potentials between one extremity of this interpolated resistance and the remote end of the coils was observed. From this the difference of potentials between the two ends of the coils was immediately deducible.

Temp. 8°·2 C. March 11, 1870. Battery 180 cells.

Electrometer.		Electrodynamometer.		Resistance in B. A. units. $1=10^9$ centims. per second.	v in centims. per second.
Difference.	Mean.	Deflection.	Mean.		
63·3 63·1	63·2	R. 420·5 L. 421·0	420·75	15587	$293·0\times10^8$
63·4 63·4	63·4	L. 420·5 R. 420·5	420·5	(15597)	$292·2\times10^8$
63·1 63·3	63·2	L. 420·0 R. 419·0	419·5	15607	$292·3\times10^8$

Temp. 9°·4 C. March 15, 1870. Battery 180 cells.

Electrometer.		Electrodynamometer.		Resistance in B. A. units. $1=10^9$ centims. per second.	v in centims. per second.
Difference.	Mean.	Deflection.	Mean.		
61·5 61·4	61·45	R. 412·0 L. 408·5	410·25	15555	$297·0\times10^8$
61·8 62·0	61·90	R. 410·0 L. 408·5	409·25	....	$294·5\times10^8$
61·7 61·4	61·55	R. 409·0 L. 407·5	408·25	....	$295·8\times10^8$

Temp. 9°–9°·7 C. March 16, 1870. Battery 180 cells.

Electrometer.		Electrodynamometer.		Resistance in B. A. units. $1 = 10^9$ centims. per second.	v in centims. per second.
Difference.	Mean.	Deflection.	Mean.		
61·7 61·8	61·75	R. 424·0 L. 411·0	417·5	15342	$294·0 \times 10^8$
61·1 60·7	60·90	R. 420·0 L. 408·0	414·0	(15372)	$297·5 \times 10^8$
61·3 61·4	61·35	R. 419·5 L. 407·0	413·25	(15402)	$295·6 \times 10^8$
61·4 61·1	61·25	R. 417·0 L. 406·0	411·5	(15432)	$295·8 \times 10^8$
61·3 60·9	61·10	R. 416·0 L. 413·0	414·5	(15462)	$298·4 \times 10^8$
61·3 61·2	61·25	R. 417·0 L. 405·0	411·0	(15492)	$297·0 \times 10^8$
61·5 61·2	61·35	R. 417·5 L. 408·0	412·75	15522	$297·7 \times 10^8$

The resistance of the coils was measured at the beginning and at the end of this series. The intermediate values of the resistance are inserted on the supposition of a gradual increase in the resistance of the coils.

The suspending-wire having been soldered to the brass cover to which it had been attached, the period of vibration of the coil was found to be 4·789 seconds; so that, for succeeding observations, $\gamma = ·000021884$.

At this stage an attempt was made to use simultaneous measurements made by means of the old and new absolute electrometers; but this method was not carried out, chiefly on account of difficulties in the use of the old absolute electrometer arising from imperfect insulation and other causes. The former method of comparison simply with the new absolute electrometer was resumed.

Temp. 10°·8 C. March 22, 1870. Battery 180 cells.

Electrometer.		Electrodynamometer.		Resistance in B. A. units. $1 = 10^9$ centims. per second.	v in centims. per second.
Difference.	Mean.	Deflection.	Mean.		
61·15 61·3	61·22	R. 391·25 L. 391·5	391·4	15654	$293·6 \times 10^8$
61·1 60·8	60·95	R. 389·5 L. 390·0	389·25	....	$294·0 \times 10^8$

Temp. 14°·8 C. March 23, 1870. Battery 180 cells.

Electrometer.		Electrodynamometer.		Resistance in B. A. units. $1=10^9$ centims. per second.	v in centims. per second.
Difference.	Mean.	Deflection.	Mean.		
60·0 59·4	59·70	R. 372·5 L. 370·5	371·5	15617	$292{\cdot}6\times10^8$
58·7 58·7	58·70	R. 367·5 L. 367·0	367·25	15633	$295{\cdot}9\times10^8$
60·0 59·1	59·55	R. 367·5 L. 365·5	366·5		$291{\cdot}7\times10^8$
59·0 58·9	58·95	R. 364·0 L. 364·0	364·0	15658	$293{\cdot}6\times10^8$
59·0 59·0	59·00	R. 363·75 L. 362·75	363·25	15673	$293{\cdot}5\times10^8$
59·0 59·0	59·00	R. 361·0 L. 361·5	361·25		$293{\cdot}0\times10^8$

The figures in the electrometer and electrodynamometer columns of the above Tables represent the readings in the arbitrary divisions of the two instruments. To deduce the value of v, these readings were reduced to absolute measure by means of the expressions for $V-V'$ and γ already given. The resistance introduced into the circuit being equal to that of the coils, half the difference of potentials indicated by the electrometer was employed in the calculations.

III. The instruments having been removed to the new laboratory, were again set up in the spring of 1871, but no observations were made till September following.

When the electrometer was set up, a series of weighings was gone through to determine the displacement of the movable disk produced by a given force. A long series of weighings of May 15th and 16th gave 200·19 micrometer-divisions as the displacement due to ·6 gramme. Before the electrometer was again used in September a longer series of weighings was gone through: this gave as the mean value of the displacement 200·55 divisions, and this value was adopted in succeeding measurements.

The electrodynamometer was set up in the same manner as formerly; the only alteration was in the manner of adjusting the lamp and lens, which were now supported and adjusted in the manner already described. A new determination was made of the moment of inertia of the suspended coil: this had become necessary from an alteration in the weight of the coil arising from the loss of a part of the mirror.

The moment of inertia of the vibrator with which the coil was compared was 4293·772 centimetre-grammes. The periods of vibration of the vibrator and coil were 25·939 and 15·5575 seconds respectively. This gave as the moment of inertia of the suspended coil 1544·6036 centimetre-grammes. After this comparison the mirror of the coil was resilvered, and was found to have increased in weight ·101 gramme. With this increase of weight the moment of inertia of the coil became 1544·704 centimetre-grammes. The distance of the scale was made 354·6 centims. The period of vibration of the coil was 5·823 seconds. With these alterations the value of γ became ·000020114.

Temp. 17° C. September 12, 1871. Battery 90 cells.

Electrometer.		Electrodynamometer.		Resistance in B. A. units. $1 = 10^9$ centims. per second.	v in centims. per second.
Difference.	Mean.	Deflection.	Mean.		
36·25 36·25	36·25	R. 647·5 L. 647·5	647·5	15960	$298·8 \times 10^8$
36·25 36·3	36·27	R. 646·75 L. 646·75	646·75	....	$298·5 \times 10^8$

September 16, 1871. Battery 90 cells.

Electrometer.		Electrodynamometer.		Resistance in B. A. units. $1 = 10^9$ centims. per second.	v in centims. per second.
Difference.	Mean.	Deflection.	Mean.		
35·1 35·1	35·1	R. 608·5 L. 613·5	611·0	(15960)	$299·8 \times 10^8$
35·6 34·4	35·0	R. 602·5 L. 600·5	601·5	assumed from former measurements.	$298·3 \times 10^8$
34·3 34·8 35·25	34·8	R. 616·0 L. 613·0	614·5	....	$303·9 \times 10^8$
36·6 35·4	36·0	R. 615·5 L. 614·5	615·0	....	$293·2 \times 10^8$
35·4 35·55	35·5	R. 613·5 L. 613·5	613·5	....	$297·0 \times 10^8$
35·6 35·8	35·7	R. 613·5 L. 615·0	614·25	....	$295·5 \times 10^8$

A considerable degree of uncertainty attaches to the foregoing results on account of their variations amongst themselves, and also because an independent determination of the resistance of the coils was not made. The variations are due chiefly to variations in the electrometer readings, to which there was the greater liability on account of the smallness of the difference of potentials measured. A battery of 90 cells was employed, no additional resistance having been introduced into the circuit.

In the series which follow a continuous measurement of resistance was made. The same battery was used, but the differences of readings in the electrometer show less variation among themselves. The period of vibration of the coil was 5·811 seconds, and the corresponding value of γ ·00002015.

September 22, 1871. Battery 90 cells.

Electrometer.		Electrodynamometer.		Resistance in B. A. units. $1=10^9$ centims. per second.	v in centims. per second.
Difference.	Mean.	Deflection.	Mean.		
33·45 33·3	33·37	R. 542·25 L. 544·25	543·25	15825	$295·4\times10^8$
34·0 33·85 33·95	33·93	R. 545·75 L. 542·25	544·00	15805	$290·3\times10^8$
34·0 33·8 33·3	33·70	R. 543·75 L. 545·0	544·37	15810	$292·5\times10^8$

September 24, 1871. Battery 90 cells.

Electrometer.		Electrodynamometer.		Resistance in B. A. units. $1=10^9$ centims. per second.	v in centims. per second.
Difference.	Mean.	Deflection.	Mean.		
34·25 34·0	34·12	R. 551·0 L. 550·5	550·75	15840	$291·1\times10^8$
34·0 34·1 34·0 34·0	34·02	R. 552·5 L. 551·25	551·87	15834	$292·2\times10^8$
33·75 33·9 34·0 33·9	33·88	R. 551·75 L. 550·75	551·25	15832	$293·2\times10^8$

September 25, 1871. Battery 90 cells.

Electrometer.		Electrodynamometer.		Resistance in B. A. units. $1=10^9$ centims. per second.	v in centims. per second.
Difference.	Mean.	Deflection.	Mean.		
35·5 35·25 35·2	35·32	L. 601·0 R. 602·0	601·5	15728	$291·8\times10^8$

September 26.—The period of vibration of the suspended coil was observed and found to be 5·805 seconds. The corresponding value of γ was ·000020175.

Electrometer.		Electrodynamometer.		Resistance in B. A. units. $1=10^9$ centims. per second.	v in centims. per second.
Difference.	Mean.	Deflection.	Mean.		
32·9 33·0 32·75 32·8	32·86	R. 523·0 L. 522·5	522·75	15640	$291·3\times10^8$

September 28, 1871. Battery 90 cells.

Electrometer.		Electrodynamometer.		Resistance in B. A. units. $1=10^9$ centims. per second.	v in centims. per second.
Difference.	Mean.	Deflection.	Mean.		
33·2		R. 536·5			
33·5	33·35	L. 536·5	536·5	15715	$292·0\times10^8$
32·85		L. 529·5			
32·8	32·79		528·0	15715	$294·6\times10^8$
32·5					
33·0		R. 526·5			
32·7	32·81	L. 518·25			
33·1					
32·7			518·25	15711	$291·6\times10^8$
32·8					
32·75		R. 518·25			
32·6	32·63	R. 520·0			
32·5					
32·4					
32·8			519·25	15722	$293·7\times10^8$
32·8					
32·5					
32·8		L. 518·5			

No further comparisons were made till February 21, 1872. In the series of observations then taken, to secure a greater difference of readings in the electrometer, a larger battery was employed, and a resistance of 10,000 ohms was introduced into the circuit. The difference of potentials between the ends of the coils was thus $\frac{R}{R+10,000}\cdot e$, where e represents the difference of potentials read off in the electrometer, and R is the resistance of the electrodynamometer circuit.

The scale was removed to a greater distance, 367·9 centims., and the period of vibration of the coil was 5·819 seconds. The corresponding value of γ was ·000019761.

The resistance of the coils at the observed temperature was afterwards measured and found to be 15834 B. A. units. This value of R is used in the calculations.

Temp. 15° C. February 21, 1872.

Electrometer.		Electrodynamometer.		Resistance in B. A. units. $1=10^9$ centims. per second.	v in centims. per second.
Difference.	Mean.	Deflection.	Mean.		
		R. 630·5	632·00	15834	$293·4\times10^8$
58·0	58·00	L. 633·5			
58·1	58·10	L. 636·0	634·75	....	$293·5\times10^8$
58·1		R. 633·5			
		L. 638·25			
58·2		R. 634·5	636·19	....	$293·1\times10^8$
	58·25	L. 634·0			
58·3		R. 634·0			
		L. 638·0	636·12	....	$292·3\times10^8$
58·4	....	R. 634·25			
		R. 635·0	636·62	....	$293·5\times10^8$
58·2	....	L. 638·25			
		L. 638·5	636·75	....	$293·5\times10^8$
58·2	....	R. 635·0			

Determination of the Area of the Movable Coil.

Before unwinding the coils for the purpose of measuring the length of the wire, and counting the number of turns in each coil, a comparison of the large coils and the suspended coil was made to determine the magnetic moment of the latter. This quantity, which appears in the calculations as $i\pi c^2$, could not be obtained very accurately by measurement of the outer and inner radii of the coil, seeing that the difference between these radii was very considerable in comparison with either.

The method of comparison followed was suggested by Professor CLERK MAXWELL. The simultaneous action of the large coils and the smaller coil upon a magnet placed in their neighbourhood was observed. The distances between the coils and the magnet being so arranged that the simultaneous action of the opposing coils upon the magnet was zero, and these distances being known, the ratio between the areas of the coils was easily obtained. The chief difficulty being to measure accurately these distances, two symmetrical observations were made with the smaller coil on opposite sides of the magnet. The distance between the positions of the same part of the coil in these two observations was measured, and half this distance taken as the distance of the centre of the coil from the magnet in each position.

The coils (see Plate XXXII. fig. 5) rested upon a narrow table (t) which had been prepared for this comparison. A V-groove (v) ran from end to end of it parallel to its edge, near one side, to guide the adjustment of the magnet and smaller coil. The stands (s, s') to which the large coils had been attached were placed at opposite ends of the table, and were so adjusted that the line joining the centres of the coils passed through the centre of the magnet.

The magnet (m), which had been used as a declinometer, was suspended by a long fibre in an enclosed space. It carried a light galvanometer-mirror, whose motions were visible through a small lens, which formed part of the glass front of the instrument. The declinometer, two of its feet resting in the V-groove, was placed between the coils, so that the centre of the mirror or magnet was equidistant from the centres of the large coils. The "suspended" coil, supported on a wooden stand (p), was placed between the magnet and one of the large coils, its motions to and from the magnet being guided parallel to itself by the groove in which two of the feet of the stand rested: it was adjusted so that its centre also should be in the line joining the centres of the large coils. The three coils were arranged so as to have their planes parallel, and so that the action of the two large coils should conspire to oppose the action of the smaller coil, which was placed much nearer the magnet. The motions of the magnet were observed in the usual manner, by reflection from the mirror. A current from a battery of 120 cells was made to pass through the three coils in series. The smaller coil was moved to and fro parallel to itself until no deflection of the magnet could be detected. During this observation it was found that the magnet was sensitive to the smallest motion of the coil that was appreciable.

The position of the coil with reference to the table was noted by marking on the table the position of the front and of the back of the stand.

The coil was then transferred to the other side of the magnet, and a similar observation made. The distance between the two positions of the coil was found to be 42·221 centims. Accordingly 21·11 centims. was taken as the distance of the centre of the smaller coil from the magnet in each observation. The distance between the centres of the large coils, measured directly, was 131·48 centims.; so that the distance of the centre of each from the magnet was 65·74 centims.

Calling these distances a and b respectively, we have, for zero deflection, the following equation :—

$$\frac{2\pi nr^2}{(r^2+b^2)^{\frac{3}{2}}}\cdot\mathrm{F}+\frac{2\pi n'r'^2}{(r'^2+b^2)^{\frac{3}{2}}}\mathrm{F}'=\frac{2i\pi c^2}{(c^2+a^2)^{\frac{3}{2}}}.$$

This is a cubic equation in c^2, and the solution of it would yield c^2 in terms of the known quantities n, n', r, r', a, b, &c. Two of the roots of this equation are imaginary, the coefficient of c^2, when the equation is reduced to the typical form, $x^3+qx+r=0$, being positive.

Without going through the ordinary process of solution to find the third root of this equation, which from the complicated form of the coefficients in this case is not convenient, c can be readily found by the following method of approximation. c is approximately known by measurement, and i, the number of turns of wire in the coil, is also known. Assuming this value of c in $(c^2+a^2)^{\frac{3}{2}}$, $2i\pi c^2$ is given by the equation. Substituting the value of i in this result, a new value of c is obtained. Using this new value of c, by means of the equation we obtain a new value for $2i\pi c^2$, and so on through several approximations.

Thus, starting with $c=4{\cdot}3$ centims., we find for $i\pi c^2$ the successive values 195681, 195435, 195421, 195420 square centims., with the corresponding values of c 4·25427, 4·2516, 4·25144, and 4·25143.

Starting with a value of c on the other side of this limit, $c=4{\cdot}25$, we find for $i\pi c^2$ 195412, 195419, with the corresponding values of c 4·25135, 4·25143. The value of $i\pi c^2$, 195420, has been adopted in the calculations.

When the comparison of the coils had been completed, the wire was unwound from the coils. The resistance of the wire was measured before unwinding, and also after unwinding, to test for short-circuiting.

Before unwinding (temp. 13° C.) the resistance of the wire of the right coil was 5630 ohms, after unwinding (temp. 14°·1 C.) 5654 ohms, the rise of temperature being sufficient to account for an increase of resistance of about 20 ohms.

The resistance of the wire of the left coil before unwinding (temp. 11°·75 C.) was 5705 ohms, after unwinding (temp. 14°·1 C.) 5726 ohms.

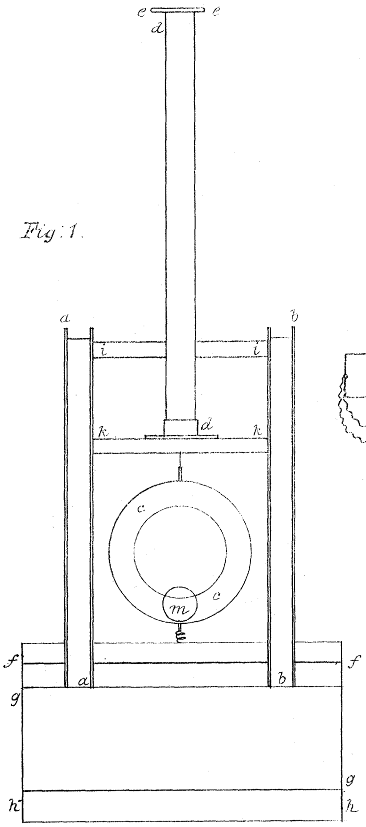
From this comparison of resistance it may be inferred that the insulation in the coils was very nearly perfect, that the effective length of the wire was that due to all the turns in each of the coils, and that the estimate of their electromagnetic effect derived from this was as nearly as possible accurate.

The mean value of v given by all the series of observations of 1870, which include, however, considerable variations, is $294 \cdot 5 \times 10^8$ centims. per second. The mean result of the later comparisons, excluding those which have been mentioned as less trustworthy, is $292 \cdot 4 \times 10^8$ centims. per second.

These later observations were made under much more favourable circumstances; still, as the causes of the variations in the earlier comparisons are not known, it has been thought right to include them also in the record of the observations.

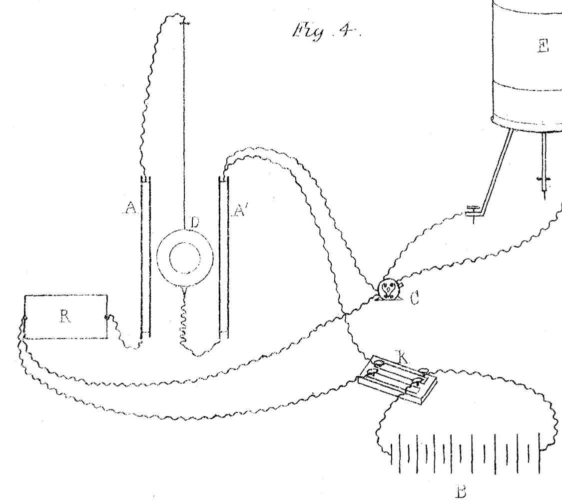
The last measurement, that of 1872, furnishes the most probable value of v, 293×10^8 centims. per second.

Fig: 1.



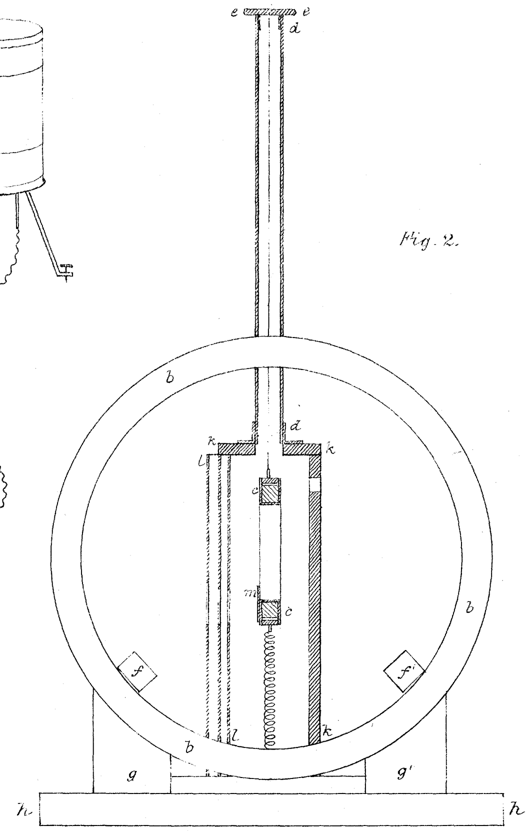
Electrodynamometer-Front elevation.
Front of Box removed to shew Suspended Coil.

Fig. 4.



A. Right Fixed Coil. E. Electrometer.
A'. Left " " C. Electrometer Key.
D. Suspended. " B. Battery.
R. Resistance. K. Battery Key.

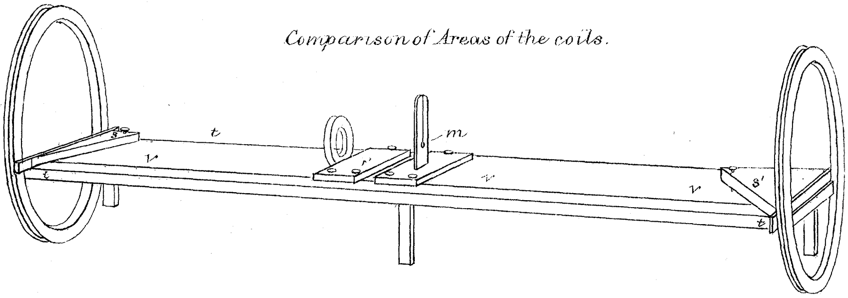
Fig. 2.



Electrodynamometer Side elevation (Box & Suspended Coil shewn in section.)

Fig. 5.

Comparison of Areas of the coils.



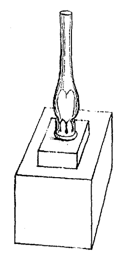


Fig. 3.